what is appropriate mathematics education for four-year-olds?

pre-kindergarten teachers' beliefs

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ABSTRACT

This study explored pre-kindergarten teachers' beliefs about the appropriateness of early mathematics education. Thirty pre-kindergarten teachers of four-year-olds, half working with low-SES children at publicly funded pre-kindergartens and the other half with middle-SES children at private pre-kindergartens, were interviewed concerning written vignettes describing two fictitious pre-kindergarten teachers' contrasting pedagogies concerning key issues in teaching mathematics to young children. The low-SES publicly funded pre-kindergarten teachers tended to support a strong focus on goal-based mathematics teaching at pre-kindergarten and at home to get children ready for kindergarten. The middle-SES private pre-kindergarten teachers tended to endorse flexible mathematics education relying on a child-centered curriculum and child-initiated learning and to oppose the instructional use of computers. Both groups, however, were similarly likely to mention that with increased academic demands, pre-kindergarten teachers needed to provide mathematics education, especially in simple arithmetic, albeit in a fun manner, without triggering stress or anxiety. These findings have significant implications for professional development.

KEYWORDS *mathematics education, pedagogy, pre-kindergarten, teacher beliefs*

introduction

Never before has so much attention been paid to early mathematics education prior to kindergarten. The National Council of Teachers of Mathematics (NCTM),

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for the first time, included the pre-kindergarten level in its revised standards for school mathematics based on the acknowledgement that '[t]he foundation for children's mathematical development is established in the earliest years' (NCTM, 2000: 75). Subsequently, the National Association for the Education of Young Children (NAEYC), jointly with the NCTM, issued a position statement that advocates 'high–quality, challenging, and accessible mathematics education for 3- to 6-year-old children' (NAEYC and NCTM, 2002: 1). As a result, nearly all states have incorporated pre-kindergarten level mathematics in their standards and curriculum framework (NAEYC, 2003; NAEYC and NAECS/SDE, 2002; Neuman and Roskos, 2005).

The impetus for the current national and state-level initiatives for effective pre-kindergarten mathematics education is two-fold. One factor is a serious concern about American students' difficulties with mathematics. Nationwide tests and international comparisons have consistently shown that many American students are falling short of the mathematical proficiency required for an increasingly mathematical and technological society (National Research Council, 2001). Noticeably, while the gap between American children and their counterparts in China, Japan, and Korea is slight in four-year-olds (Case et al., 1996; Ginsburg et al., 1997), it becomes more significant by kindergarten (Stevenson et al., 1986) and first grade (Stevenson et al., 1990; Zhou et al., 2005) and certainly by fourth grade (US DOE and NCES, 1997).

The other and more promising factor is the research-based awareness that young children already posses well-developed informal mathematics (Ginsburg et al., 1998), are ready to learn complex mathematics (Greenes, 1998), and thus, 'more can be learned in the preschool years than was previously understood' (Bowman et al., 2001: 9). This recent understanding is a big step forward since traditionally, mathematics was mentioned primarily in the context of cautions and warnings (e.g. Elkind, 1981, 1998) that purposefully teaching it is unnecessary, inappropriate, or even harmful to young children (Balfanz, 1999). This type of belief was based on a limited understanding of children's mathematical learning and development mainly drawn from Piaget's theory of pre-operational children's cognitive immaturity (Ginsburg et al., 2001; Walsh, 1991). Yet, a large body of research since Piaget shows that young children are more competent than previously assumed and suggests that inadequate attention to mathematics in early childhood may lead to later school failure, especially for children from poor and minority families, who are less likely to have a home environment in which their academic learning is facilitated (Bowman et al., 2001; Lee and Burkam, 2002).

Now that the leaders in the field have endorsed a new vision of effective early mathematics education, its realization depends greatly on pre-kindergarten teachers, as they are the ones who ultimately determine what is implemented in their classrooms. The recommended new practices will be filtered through the lenses of the teachers' personal beliefs (Pajares, 1992; Richardson, 1996),

which may in turn affect their classroom practices (Charlesworth et al., 1993; Stipek and Byler, 1997). Any attempts to change or improve classroom practices without consideration of the teachers' beliefs have often resulted in superficial change fraught with misunderstanding and misinterpretation (Kostelnik, 1992; Ryan, 2004; Walsh et al., 1993), or even worse, in strong resistance to change (Bailey, 2000; Sarason, 1990, 1996). Therefore, with the current focus on early mathematics education, it is more important than ever to understand pre-kindergarten teachers' beliefs in order to achieve a high quality mathematics education in pre-kindergarten classrooms.

Yet, pre-kindergarten teachers' beliefs about teaching and learning of mathematics have remained virtually unexplored (Einarsdottir, 2003; Feiler, 2004; Genishi et al., 2002). Existing studies typically use questionnaires or surveys to capture pre-kindergarten teachers' beliefs regarding school readiness or classroom practice. The studies tend to categorize beliefs according to broad theories of development such as maturationist, behaviorist, or interactionist (e.g. Caruso et al., 1992) or according to general notions such as developmentally appropriate versus developmentally inappropriate practice (e.g. Snider and Fu, 1990), or child-oriented versus skill-oriented approach (e.g. Stipek and Byler, 1997). These types of studies, however, can provide, at best, only a general, abstract sense of teachers' beliefs, and furthermore run the risk of misrepresenting them (Kagan, 1990) as in reality teachers' beliefs may be more complex than these general notions suggest (Genishi et al., 2002).

Stipek and Byler (1997), for example, examined teachers' beliefs regarding early childhood practices in general and found that pre-kindergarten, kindergarten, and first grade teachers of low-socio-economic status (SES) children, compared to those of middle-SES children, tended to agree more with a skills-oriented approach that emphasizes academics learning through structured, teacher-directed instruction and carefully sequenced tasks, repetition, practice, as opposed to a child-oriented approach that stresses social and emotional development through child-initiated, open-ended exploration and discoveries. While these results are noteworthy, belief differences between teachers of low- and middle-SES children should be examined in greater depth so as to provide a finer grained analysis that might yield valuable information for the design of effective professional development to help teachers develop their thinking and practices (Richardson, 1996).

In order to achieve this goal, our study employed a relatively unique method of using written vignettes to elicit teachers' beliefs. The participating teachers were presented with a written account of two hypothetical teachers' very different responses to an educational dilemma. Using a qualitative interview method, we ask the teachers to evaluate the responses described in the vignettes and to present their own views. This method is an attempt to contextualize and concretize controversial issues relating to early mathematics education. Our study, which is exploratory and descriptive in nature, aimed to examine pre-kindergarten

teachers' beliefs about appropriate mathematics education for four-year-olds and to determine whether teachers' beliefs are related to the SES of their young students.

methods[1]

participants

The participants in this study were 30 New York City pre-kindergarten head or group teachers – a half working with low-SES children and the other half working with middle-SES children – who taught classrooms in which most of the children were four-year-olds. The researchers sent out solicitation letters to directors and teachers at public pre-kindergarten programs fully funded by the federal government (e.g. Head Start) and/or by New York State (e.g. Universal Pre-Kindergarten), which give preference to children from low-income families, to recruit low-SES teachers, and to those at private pre-kindergarten programs relying solely on tuition received from parents to recruit middle-SES teachers. The first 30 pre-kindergarten teachers who volunteered to participate in the study were selected.

The low-SES teachers indicated that all of their students were enrolled in a free lunch program, and that the majority of their students were Black (on average 42.47%) and Hispanic (50.03%), while the middle-SES teachers indicated that the majority of their students were White (78.20%). Also the low-SES teachers had significant higher numbers of English Language Learners than middle SES teachers (31.62% vs 7.63%, $F = 7.40$, $p < 0.05$) (see Table 1 for characteristics of student population).

These two groups of pre-kindergarten teachers differed in terms of their own background characteristics. The low-SES teachers tended to be from a minority

table 1 characteristics of student population by SES

		Low-SES pre-kindergartens	Middle-SES pre-kindergartens
		Mean number (%) of students	Mean number (%) of students
Class size		18.53 (100.00)	19.27 (100.00)
Ethnicity**	White	0.47 (2.54)	15.07 (78.20)
	Asian	0.27 (1.46)	1.86 (9.65)
	Black	7.87 (42.47)	0.86 (4.46)
	Hispanic	9.27 (50.03)	1.21 (6.28)
	American-Indian	0.13 (0.70)	0.00 (0.00)
	Others	0.33 (1.78)	0.43 (2.23)
English Language Learners*		5.86 (31.62)	1.47 (7.63)

*$p < 0.05$. **$p < 0.01$.

background ($X^2 = 17.23$, $p < 0.01$), have a lower level of education ($X^2 = 7.19$, $p < 0.05$), and shorter pre-kindergarten teaching experience ($F = 4.74$, $p < 0.05$) than the middle-SES teachers (see Tables 2 and 3 for background information on the participating teachers).

vignettes

A series of 10 written vignettes, which illustrated salient or challenging issues facing the early childhood education field as it attempts to implement the NAEYC/NCTM standards, was developed to stimulate pre-kindergarten teachers to express their pedagogical beliefs regarding early mathematics education. The topics depicted in these vignettes included 1) purpose of pre-kindergarten education, 2) pre-kindergartners' readiness for mathematics education, 3) emergent and planned curriculum, 4) integrated and separate subject matter curriculum, 5) promotion of knowledge/skills and dispositions/feelings, 6) teacher-directed, formal and child-initiated, informal activities, 7) children's individual differences

table 2 participants' ethnicity, final degree, and specialization by SES

		Low-SES pre-kindergartens	Middle-SES pre-kindergartens
		Frequency (%)	Frequency (%)
Ethnicity**	White	2 (13.3)	11 (73.3)
	Asian	1 (6.7)	3 (20.0)
	Black	8 (53.3)	0 (0.0)
	Hispanic	3 (20.0)	1 (6.7)
	Others	1 (6.7)	0 (0.0)
Final degree*	Associate's degree	2 (13.3)	0 (0.0)
	Bachelor's degree	9 (60.0)	4 (26.7)
	Master's degree	4 (26.7)	11 (73.3)
Specialization	Early Childhood Education	9 (60.0)	11 (73.3)
	Others	6 (40.0)	4 (26.7)

* $p < 0.05$. ** $p < 0.01$.

table 3 participants' teaching experiences of pre-kindergarten and upper grade by SES

	Low-SES pre-kindergartens		Middle-SES pre-kindergartens	
	Mean	(SD)	Mean	(SD)
Pre-kindergarten teaching*	6.40	(3.83)	11.33	(7.90)
Upper grade teaching	2.50	(7.66)	2.40	(4.69)

* $p < 0.05$.

in interest, 8) children's individual differences in competence, 9) use of concrete materials and computers, and 10) parents and home environment. Each vignette described the contrasting pedagogies of two pre-kindergarten teachers, whose practices exemplified polar-end stances – skills-oriented at one end and child-oriented at the other. Four early childhood experts, besides the two authors, reviewed the vignettes to ensure that they, indeed, reflected issues important in early mathematics education (see Appendix A for a sample vignette). It was expected that evaluating the appropriateness of two very different approaches to key issues of pre-kindergarten mathematics education would lead teachers to reveal their beliefs concerning appropriate or even ideal pre-kindergarten pedagogy.

procedure

The first author interviewed each of the 30 participants individually concerning the vignettes. The interviewer asked the participant to draw on her own experiences as a teacher of four-year olds to evaluate the issues raised by the vignettes. After reading each vignette, the participant was asked the following open-ended questions:

- If you were facing a similar situation in your own classroom, which of these two teachers' practices would you favor? This does not have to be an either/or question; you may also support practices of both teachers or neither.
- What is it about the first/second teacher's approach that makes it good/not so good for the children in your classroom?
- What would you recommend as a best practice to these two teachers if they were working with your students?

The individual interview sessions took approximately one hour. All interviews were audio-taped and later transcribed.

data analysis

The data analysis of this study, which involved two phases, was guided by a grounded theory approach of generating relevant codes and dimensions from the collected data (Strauss and Corbin, 1990).

In the first phase, using NUD·IST, a software program that facilitates the organization and analysis of qualitative data, the transcripts of the 30 participants' comments were segmented into relevant *beliefs* expressed by the teachers. Among a wealth of beliefs produced by the 30 participants, the beliefs that were shared by at least 20 per cent of the participants were identified as *codes* for further analysis. A total of 61 codes were identified. For each interview transcript, each of the

10 vignettes was examined to determine whether any of the 61 codes was present, and the frequency for each code was calculated by the number of vignettes in which the code was present. The frequency ranged from 0 to 10 with 0 indicating *absent* in all of the 10 vignettes and 10 indicating *present* in all of the 10 vignettes. Thus, a teacher who repeatedly insisted in responding to many vignettes that ready-made mathematics curricula, for example, are desirable would receive a higher score on this code than would a teacher who never mentioned ready-made mathematics curricula. Inter-rater agreement was established between the first author and a trained graduate student for 15 of the 30 interview transcripts by calculating Cohen's kappa for each of the codes, which ranged from 0.82 to 0.98, which indicate statistically acceptable levels of agreement.

In the second phase, as the qualitative data from the interview transcripts were converted into quantitative data of frequencies, SPSS, a software program for the statistical analysis of quantitative data, was used to conduct further analysis of the 61 codes to extract meaningful *dimensions* underlying the teachers' beliefs. First, codes that appeared to be related conceptually were grouped together if they were also strongly and significantly correlated to each other. Then, principal components analysis (PCA) was performed on the codes classified as one group. A group of selected codes was determined to be a meaningful dimension when only one factor with an eigen-value greater than 1 was extracted by PCA. During this process, 17 of the 61 codes, which had too little variance or that did not associate significantly with other codes, were dropped. This procedure identified nine dimensions underlying the pre-kindergarten teachers' beliefs about early mathematics education.

Strong focus:	Teachers' strong focus on mathematics education for kindergarten preparation
Flexible:	Teachers' endorsement of mathematics education flexible to meet children's other needs and interests
Goal-based:	Teachers' endorsement of mathematics education based on the teachers' goals and plans
Child-centered:	Teachers' endorsement of mathematics education centered on children and their interests
Arithmetic:	Teachers' endorsement of mathematics education focusing on numeracy and arithmetic through direct instruction
Child-initiated:	Teachers' endorsement of mathematics education promoting positive dispositions through self-exploration and problem-solving
Enjoyable:	Teachers' endorsement of fun and enjoyable mathematics education for young children
Non-pressured:	Teachers' endorsement of mathematics education without any pressure or stress acknowledging individual children's ability and aptitude

No computers: Teachers' opposition to the instructional use of computers in pre-kindergarten classrooms.

Detailed descriptions and examples of these dimensions are presented in the results section. The factor score for each dimension, which was produced by PCA using the regression approach in the process of identifying each dimension, was saved for further statistical analysis. The distributions of each of these factor scores have a mean of zero and a standard deviation of one. (See Appendix B for the list and definitions of the codes, from which each dimension was derived, and the correlations of each of the codes with its dimension.)

results

In order to identify the differences as well as the commonalities between the beliefs held by the low- and middle-SES teachers, one-way ANOVAs were conducted on each of the factor scores for the belief dimensions (see Table 4 for the ANOVA results). The results revealed that among the nine dimensions, the low-SES teacher group scored significantly higher on two dimensions, *Strong Focus* and *Goal-Based*; the middle-SES teacher group scored significantly higher on four dimensions, *Flexible, Child-Centered, Child-Initiated,* and *No Computers*; and the two groups did not differ on the remaining three dimensions, *Enjoyable, Non-Pressured,* and *Arithmetic*. These commonalities and differences between the two groups' beliefs regarding appropriate early mathematics education are described below using their quantitative scores on the dimensions as well as qualitative interview excerpts to provide deeper insights into their beliefs.

table 4 teacher beliefs by SES

	Low-SES pre-kindergartens		Middle-SES pre-kindergartens		ANOVA results
Belief dimensions	Mean	(SD)	Mean	(SD)	*F*
Arithmetic	–0.05	(1.17)	0.05	(0.80)	0.06
Enjoyable	0.05	(1.17)	–0.05	(0.84)	0.08
Non-pressured	–0.02	(0.82)	0.02	(1.18)	0.02
Strong focus	0.56	(1.10)	–0.56	(0.43)	13.78***
Goal-based	0.40	(1.17)	–0.40	(0.60)	5.62*
Flexible	–0.38	(0.93)	0.38	(0.95)	4.90*
Child-initiated	–0.41	(0.45)	0.41	(1.23)	5.99*
Child-centered	–0.55	(0.36)	0.55	(1.14)	12.89***
No computers	–0.68	(0.56)	0.68	(0.87)	26.08***

* $p < 0.05$. ** $p < 0.01$. *** $p < 0.001$

shared beliefs of low- and middle-SES pre-kindergarten teachers

First, the pre-kindergarten teachers of low- and middle-SES children did not differ on the dimension, *Arithmetic*. Both groups, to a similar extent, mentioned the pressure of current educational trends which demand early mastery of mathematics, and addressed the importance of teaching arithmetic, including one-to-one correspondence, counting, simple addition and subtraction, and writing numerals, and the need for a certain degree of direct instruction to teach young children.

> Nowadays, school is much different than when I went to school. Children are expected to know more. [...] I have parents that are sending their children to private schools. They have to meet certain academic requirements. The public schools are requiring more and more preschool knowledge. Most kids are expected to go to preschool.

> Math is not just counting or to be able to add or subtract. At an early age, being able to do one-to-one correspondence. 'Cause my children will know how to count but they don't know how to match. That's part of math also.

> As the year goes on, we do very basic addition and subtraction where we talk about if there are only, you know, there's supposed to be 20 kids, and only 18 kids show up. How many kids are not here?

> They see numbers in the calendars, they see numbers everywhere in the streets and their house. So they need to be exposed to numbers. Add numbers to what she [Teacher D in the vignette] is teaching. 'Look this is the number 5. These are five birds. This is the number 5.' I've done that.

> There needs to be a time when it is teacher-directed because then, how are the kids to learn? The kids need to learn on their own. But the kids need to learn from an adult as well.

Second, both groups of teachers did not differ on the dimension, *Enjoyable*. They tended to emphasize that pre-kindergarten teachers should ascertain that mathematical teaching was fun and exciting involving interesting toys or materials and conducted in small groups with teachers being sensitive to children's overall emotional well-being. They commented:

> You can have the academics in the program as long as it's fun. Keep in mind that these children are four-years old. I like when everything is fun and gameful. Make your math activities fun, you know, by playing games.

> Children really love having this gross, disgusting looking monster, a thing that we kind of forbid in the classroom. But if they sneak it in for an academic thing, they know that they've gotten it across to measure something gory and disgusting, they remember it much more. They'll remember, 'I weighed it, I counted it.'

> The kids need to have the attention of the teacher sometimes on a one-to-one basis or in a small group. [...] You do need to do an activity that is more focused to see the children, you know, and to give them the proper attention, you know, the one-to-one or the small group.

> Intellectual growth doesn't really come unless the emotional growth has accompanied it. [...] Basically, a happy, secure child will learn better than the child who feels completely overwhelmed.

Third, both groups did not differ on the dimension, *Non-Pressured*. The participating pre-kindergarten teachers tended to stress that teachers should not push children nor provoke anxiety and stress while trying to teach mathematics, especially since some four-year olds lack readiness as children have different interests, aptitudes, and ability in mathematics. They explained that when children show interest in mathematics, it was a good time to teach them the related concept.

> I think with young children, it's very true that they develop at their own rates; they have their own pace. You can't – really that goes with anything, not just their learning math – you can't force them to be ready for that. How can you accelerate their learning? [...] I think teacher P needs to not be in such a rush to try to make sure that child catches up to their peers, because chances are, that's not gonna happen

> It is fine, if the children are capable of handling it and if they're not stressed out [...] If she [the teacher P] sees that it does provoke anxiety and stress, she should stop. Maybe to put it away for a while, bring it out again [...] I have seen the aftermath of pressuring children; it's not a good one.

> She has to understand that you may teach something, and yet, some children just naturally and physically are not ready for something. [...] Many children just aren't ready. I say with hand-to-eye coordination, things like that. Again, there are many four-year olds who have a very hard time just taking a pencil and writing a representational drawing and things like that.

> There are years where I've had kids who are really into math, and it could be just because there's one or two kids and they are the leaders. And then, there are years where I've had kids really into language skills.

> Certain children just have graphic skills and very often the children who are extremely verbal don't have particular graphic skills. Sometimes children have it all. [...] If you neglect to recognize that some children have more skills than others, naturally, and you just expect them to all learn and if you're judging children on their innate talent, then that's just wrong.

> When the children say, 'Oh, look! Doesn't this remind you of the circles or this and that?' Then, it's the perfect time to return to the circles, no matter what you had planned that week. You return to the circles and you support the children who suddenly have enthusiasm for something and you see how far you could expand.

beliefs of low-SES publicly funded pre-kindergarten teachers

In addition to the shared beliefs described above, the low-SES teachers held the following distinctive pedagogical beliefs. First, compared to middle-SES teachers, they scored significantly higher on the dimension, *Strong Focus* ($M = 0.56$ vs $M = -0.56$, $F = 13.78$, $p < 0.001$). They tended to believe much more strongly

than did middle-SES teachers that mathematics education should be a priority in pre-kindergarten especially to prepare the children for kindergarten and beyond. They also tended to prefer to set time aside specifically for mathematics learning and to adopt ready-made mathematics curricula and materials. Without such preparation, they felt, their students would have a difficult time later when learning school mathematics. They explained:

> I think it's important, you know, necessary math experiences in preschool. I think that's very, very, very important. [...] Just to keep in mind that, you know, just working in my population right now, children are so disadvantaged that ideally stressing academics more than other stuff may be appropriate.

> Preschool means to be ready for school in my opinion. [...] If they have the math readiness, any kind of academic readiness that sets them apart from any other kindergartners that's coming in is also good. [...] It just puts them ahead of the game.

> Teacher K [in the vignette], I agree with, you know, having a certain specific time to work on the [math] concepts. [...] You have to have more of a specific time with them to go over concepts.

> I think that a math curricula and materials can help a teacher introduce math. It's got to be a teacher-friendly, child-friendly math curriculum that teachers can begin to use in the classroom to help young children learn and also help to broaden their ideas for math because there are some people who want to teach these little four-year-olds math, but just don't know how to do it.

> Most humans do not eventually become math literate. A lot of grown-ups I know have problems with math 'cause I know that math is difficult for me at this age. [...] So when you're exposed to these things earlier, it doesn't become so hard, but as you get older it gets harder.

Second, the low-SES teacher group scored significantly higher on the dimension, *Goal-Based*, than the middle-SES teacher group ($M = 0.40$ vs $M = -0.40$, $F = 5.62$, $p < 0.05$). They believed that teachers should work with overall goals and plans for mathematics education and organize mathematics as well as other subject matters around a theme or topic. Even when children show little interest, they felt, teachers should encourage children to engage in these activities and also encourage parents to work with their children at home to prepare and supplement school learning. These pre-kindergarten teachers commented:

> There have to be established goals and objectives. These things have to be set so you could see where a child is now and furthermore to see how much the child has accomplished. What are the goals you could set for the child? What kinds of improvements do you want to see?

> I really like the fact that she prepares children's activities around themes or topics that young children are interested in. [...] You could get a lot. You've got science, social studies, including math, out of just that one topic.

> Children need encouragement very much. If you see a child not interested in a certain activity or being that we're talking about math, you come up with some kind of strategy or tactic that will encourage them to make them want to learn about math.
>
> We're the ones who have to encourage the parents to work with their children at home. [...] Most parents are busy. So they depend on us a lot to help prepare their children for public school or move onto another school. Sometimes we give something to take home. Because parents don't have time, so we just try to provide parents at home something to work with their children in the evening.

beliefs of middle-SES private pre-kindergarten teachers

By contrast, the middle-SES teachers held the following distinctive pedagogical beliefs. First, the middle-SES teacher group scored significantly higher on the dimension, *Flexible*, than the low-SES teacher group ($M = 0.38$ vs $M = -0.38$, $F = 4.90$, $p < 0.05$). They tended to view children's social development as a pre-kindergarten priority, and preferred to postpone the introduction of mathematics activities when children do not seem to be interested or ready. They were confident that even if their young students were behind in mathematics at pre-kindergarten, they would be able to catch up later, although they felt pressured by parents who held high expectations and demanded more rigorous academics education. They commented:

> My advice would be to not underestimate the importance of socialization. [...] The math concepts, if we had to give up one thing, ultimately it would be those because the social piece is the piece that seems to be what we have with them at this age and what we can do.
>
> If the children are not as interested, you let it go. Maybe they'll be interested in it in a month from now. Maybe they're not ready developmentally. [...] Maybe reintroduce it in another way, at another time, when they're more ready. [...] If I feel that the children aren't ready for something, I'll do it at another time, another day, or I'll put off for another month or two.
>
> I don't believe that if you don't have really good math competence in preschool that you'll suffer terribly in the future, that's just too much. Children do catch up, and they do blossom at different times in their lives. [...] I have seen children that don't seem to be doing very well in this area or not, and then they end up going to Harvard, Yale, and Princeton. So you really can't tell. You have to give children time to mature.
>
> I think it largely depends on the demographic and area in which you're teaching. Here, because the parents come from, they're very educated and they're all lawyers and doctors, they want their children to be able to read before the age of five, which is, I mean, it's ridiculous, but that's what they want. [...] We use the academics almost as an icing on the cake, kind of, to make the parents feel good that they're actually doing things.

Second, the middle-SES teachers scored significantly higher on *Child-Initiated* than the low-SES teachers ($M = 0.41$ vs $M = -0.41$, $F = 5.99$, $p < 0.05$). These

teachers tended to believe that in order to develop good dispositions and feelings toward mathematics, children should learn mathematics through self-initiated play, exploration, discovery learning, and problem-solving. They were also likely to be against formal mathematics education, specifically for kindergarten preparation, as well as against the use of ready-made mathematics curricula and materials. They commented:

> Well, I don't even want to say particularly in math, really in any area, the confidence is key. If children feel good about it, they're going to go so much further and have a more positive experience. [...] But I do imagine that if they're feeling confident, this is partly because they're meeting with some sort of success.

> It's far more important to have these materials sort of in your classroom where children are free to explore with them whenever they please and actually do the learning themselves. [...] There are many times that my children amaze me because they come up with an idea or, you know, a realization about the counting something that I never thought to use that way. [...] There's nothing more amazing than when things come from the kids themselves.

> 'Do you like soccer or football? Do you like dragons or dinosaurs? Do you like *Beauty and the Beast* or *the Little Mermaid*?' And the children walk around and fill in all the children's opinions. [...] They've written it all in, they've gone around socially and asked each one of the children. There's a certain amount of power and self-esteem involved with doing this sort of activity.

> So often I hear teachers talk about preparing children for the next year and that's inappropriate. You're in the present. [...] Children are advancing at different rates and you don't become obsessed with what's happening next year. [...] I believe in working with what's appropriate for the child now.

> We are professionals. We should be given that freedom instead of just getting cookie-cutter curriculum. [...] If you get caught in this idea of just doing what's given to you, you might not be able to go with the richness of children and their, like, this exuberance they have and who they are.

Third, the middle-SES teachers scored higher on the dimension, *Child-Centered*, than the low-SES teachers ($M = 0.55$ vs $M = -0.55$, $F = 12.89$, $p < 0.001$). These teachers tended to believe that while teachers should have the content goals of important mathematical concepts in their minds, they should base the curriculum on the interests of children in their classrooms and adapt the curriculum to the level and pace of individual children. They also tended to stress that children's learning should be assessed in an informal way such as observation and interviews, not in a formal way of tests or quizzes. The teachers commented:

> You just have to be careful to decide what you really think is important to expose them to and age appropriate. [...] to sort of come up with a list of concepts that are appropriate and that the children are ready to explore [...] to sort of sit down and think what do

we think is appropriate in terms of these math concepts and what do we really think is something that is worth the children's while.

Does the teacher run with the kids' interests? Based on who they are and building from there? [...] The way I run my curriculum is, as the teacher, I have like the skeleton for the curriculum, and I have some idea of what skills I'd like them to know, and um, the kids like muscle it up, you know, the kids add all the meat to it.

It feels more old world to me, you know, where teachers had goals in mind for the whole, the class as a whole, especially if we're talking about early childhood. [...] Every group that comes in is so different, and then, you take individual children, and that makes it even more complex. So, those goals pretty much end up in the wastebasket after the first month. [...] You're individualizing your program. I think that works.

I would probably have her [Teacher T in the vignette] go around for a few weeks and be with the children and even like sort of interviewing them about what they do at home and find out who's getting, who's more mathematics savvy than other kids, who can count to ten, who can't count to ten. Like ask in ways that, you know, they don't feel pressure to play and take like a poll, and find out what the percentage of kids we have that can count and recognize shapes, can feel the world, you know.

I don't have a checklist that this child can't count to fifty, red flag, you know. [...] If you test them all the time, then they're just going to say, 'Not only is this not fun, but I'm not good at it.' [...] It's hard work because it's not a worksheet and it's not a quiz, to sort of follow up, to listen to their play, to ask a question.

Fourth and last, the middle-SES teachers scored significantly higher on the dimension, *No Computers*, than the low-SES teachers ($M = 0.68$ vs $M = -0.68$, $F = 26.08$, $p < 0.001$). The middle-SES teachers tended to strongly feel that it was inappropriate to have computers as an instructional tool in the pre-kindergarten classroom as computers prevented their children from socializing with people in the classroom, and that their students from enriched home environments, unlike those from less privileged families, had access to computers at home. They explained:

I just hate computers for children this age. [...] It's just too removed, too far removed from the senses. [...] There's no thought involved. It's totally just pressing buttons. If this doesn't work right with one button, they just randomly press another button. There's no thinking, there no process involved. There's no logical analysis of anything going on there.

I think that computers tend to just block in one child at a time. I mean, maybe it'll take in two or three, doing group activity. But it kind of isolates the child. I really don't think that computers have a place in early childhood.

Because I'm in a somewhat privileged private school, I know the children are getting exposed to computers at home. [...] If I were teaching in the daycare center or even a more underprivileged area, I would simply think it might be a good idea to expose the children to computers just so they're familiar with it.

discussion

The content analysis of the participating pre-kindergarten teachers' comments expressed in response to the written vignettes revealed that the publicly funded pre-kindergarten teachers of low-SES children and the private pre-kindergarten teachers of middle-SES children tended to share some beliefs but also to differ significantly on others (see Table 5 for summary of low- and middle-SES pre-kindergarten teachers' shared and differing beliefs). In this section, we first explore possible explanations for the commonalities and the differences found in the beliefs of the low- and middle-SES teacher groups, and then discuss the implications of this study for designing professional development to help both groups of teachers achieve high quality mathematics education in their classrooms.

possible explanations for the commonalities and differences in teachers' beliefs

Why do the pre-kindergarten teachers share certain aspects of their beliefs about appropriate early mathematics education, and differ in other aspects? Although our study, which is descriptive and exploratory in nature, was not designed to *explain* or *identify* the factors causing these commonalities and differences, it may be of value to speculate about several factors that could be at play.

the commonalities in pre-kindergarten teachers' beliefs

Both groups of pre-kindergarten teachers felt the changes in the educational field that demanded early mastery of mathematics. Receptive to these changes, the teachers generally agreed that their young students *could* and *should* engage in mathematical learning in a fun and playful manner capitalizing on children's natural interests in mathematics. Notably, however, the teachers emphasized children's individual differences in mathematical interests, aptitudes, and ability and the lack of readiness in some children, and frequently expressed their concern for pressuring or hurrying children and putting them at risk for psychological damage. This kind of shared belief among pre-kindergarten teachers may not come as a surprise considering that until the late 1990s, the *'ban academics in preschool* movements, supported by many in the early childhood *establishments'* (Schickedanz et al., 1990: 11) dominated the early childhood education field. As Nelson (1999) pointed out:

> Early childhood educators have so often been warned not to push young children in 'academic' subject areas (e.g. Greenberg, 1990, 1992) that they have a right to be

table 5 summary of pre-kindergarten teachers' shared and differing beliefs

Commonalities	
• Current educational trends demand early mastery of mathematics, especially arithmetic (e.g. one-to-one correspondence, counting, simple addition/subtraction, and writing numerals). • There is a need for a certain degree of direct instruction, and teachers should teach in small groups in a fun/exciting manner with interesting materials. • Teachers should capitalize on children's natural interests in mathematics; yet children have different interests/aptitudes/ability in mathematics, and some four-year-olds lack readiness. • Teachers should not push or provoke anxiety/stress and must be sensitive to children's overall emotional well-being.	
Differences	
Low-SES pre-kindergarten teachers	**Middle-SES pre-kindergarten teachers**
• Mathematics education should be a pre-kindergarten priority, especially to prepare children for kindergarten and beyond. • Parents should be encouraged to work with their children at home to supplement school learning. • Without pre-kindergarten mathematics preparation, children may have a difficult time later. • Teachers should organize mathematics and other subject matters around a theme or topic, yet have overall goals and plans for mathematics education and set time aside for mathematics learning. • Teachers should adopt ready-made mathematics curricula and materials. • Even when children show little interest, teachers should encourage them to engage in the mathematics activities. • Computers are effective instructional tools as they promote children's understanding and learning of mathematics in pre-kindergarten classrooms.	• Social development should be a pre-kindergarten priority; mathematics education for kindergarten is unnecessary. • Parents hold high expectations and demand more rigorous academic education in pre-kindergarten. • Even if children were behind in mathematics at pre-kindergarten, they would be able to catch up later. • Teachers should base the curriculum on the children's interests and adapt it to the level/pace of individual children, yet have content goals of mathematical concepts in mind. • Teachers should not use ready-made mathematics curriculum and materials. • When children do not seem to be interested or ready, teachers should postpone the introduction of mathematics activities. • Children should learn mathematics through self-initiated exploration, discovery and problem-solving, and develop positive dispositions/feelings towards mathematics. • Teachers should assess in an informal way (e.g. observation and interviews), not through tests and quizzes. • Computers are inappropriate instructional tools as they prevent children from socializing in the classroom and, moreover, children have access to computers at home.

> suspicious when someone advises them to proceed aggressively in that most academic of areas, mathematics. (p. 135)

In a similar vein, the pre-kindergarten teachers showed a somewhat hesitant attitude toward holding high expectations and challenging young children in mathematics.

Furthermore, mathematics activities that both groups of teachers mentioned as examples during their interview were those dealing typically with basic arithmetic, such as one-to-one correspondence, simple addition and subtraction, and number symbols or numerals. Early childhood teachers' narrow conception of mathematics, according to Copley (2004), is due to inadequate teacher preparation. Most pre-kindergarten teachers continue to focus on the simple arithmetic that they experienced as students in school (Ball, 1988, 1990a, 1990b; Lortie, 1975), as few of them had an opportunity in their teacher preparation to develop a broader understanding of appropriate mathematical content for young children (Copley, 2004; Ginsburg et al., 2006; Sarama and DiBiase, 2004). Most early childhood teacher training institutions require their teacher candidates to take only one course in mathematics, compared to several courses in language and literacy (Ginsburg et al., 2006).

the differences in pre-kindergarten teachers' beliefs

What is responsible for the strong differences between the beliefs of teachers of low- and middle-SES children about appropriate early mathematics education? We can speculate that several factors could be at play, namely, the children, the school settings, and several aspects of the teachers' own background experience.

One factor involves the children: children from low- and middle-SES families differ in many ways, including their educational needs. Children from poor and minority families show poor mathematics achievement as early as in kindergarten (Denton and West, 2002; Lee and Burkam, 2002; Stipek and Ryan, 1997). Given this unfortunate fact, perhaps teachers of low-SES pre-kindergartners are being entirely realistic when placing a high priority on their students' mathematics preparation for kindergarten and encouraging children to participate in mathematical activities regardless of their interests in order to avoid setting their students on a trajectory of underachievement in mathematics. While these teachers emphasized the importance of parental involvement at home, they also lamented the lack of it in their students' home environment. The teachers of low-SES children commented:

> A lot of parents are working parents. They are very concerned about their children's learning, they are. But after working so hard all day sometimes they're tired and they might not have enough time to maybe sit down really with their children to do, you know, to improve or to work on their math skills.

> But there are some parents who feel that children are going to learn everything they need to learn in school, and that's not always the case. Those children who really excel are those children that come with the pre-knowledge from home and other different experiences.

The teachers of middle-SES children, by contrast, generally do not need to worry as much as the teachers of low-SES children do; their students from middle-SES families are more likely to enjoy advantages and resources available in their home environment, and thus, most of them are fairly competent and knowledgeable (Delpit, 1995; Schickedanz et al., 1990). The following are typical comments from the middle-SES teachers:

> As far as I'm concerned, almost every year, but especially this year, my children are very knowledgeable. They're exposed to a lot of experiences: they're taken on trips; they're taken to museums; they have their own library cards; they have books; they have games. The parents are interested and focused on their children's education.

> My kids come from advantaged backgrounds. They do have puzzles; they do have blocks; they do have toys at home. The kids really, they have all the ideal, I guess, advantages. They have parents who read to them, parents who are able to stay at home with them, parents who have the skills to do this. So, you could say they're more sophisticated than maybe the average four-year-old.

Some middle-SES teachers rather expressed concerns about the *hot-housing* phenomenon of parents putting too much academic pressure on their young children at home (Elkind, 1981, 1987; Gallagher et al., 1987; Rescorla et al., 1991).

> The parents here, they value education. So I'm sure they do work with their children at home. I'm hoping that they don't overdo it. [...] There are parents that are high-strung about their children in terms of learning. So there are going to be parents that are like that. But, hopefully, they'll understand the importance of children being children.

> When it comes to the academics, I think parents are much better off leaving it to the teachers. I don't think they should be prepared at home. [...] The parents are really, really worried about how their children are going to fare. This is a private school, and they're applying to private schools for kindergarten. The competition is so, so stiff. The parents are all nervous and this passes to the children.

In response to this type of home environment of their middle-SES students, the teachers believed that pre-kindergarten should be a place mainly for social enrichment. In short, teachers' beliefs may (quite sensibly) reflect the realities of their young students' lives.

Another factor involves the school settings where the participants taught. The teachers of low-SES children worked at publicly funded pre-kindergartens, funded by the federal government or New York State, which may have been greatly influenced by the current 'no child left behind act' and the standards-based

accountability movement which has required strong focus on pre-kindergarten mathematics education (Bredekamp, 2004). The Head Start Child Outcomes Framework, for example, includes mathematics (Head Start Bureau, 2001). New York State has also adopted standards for mathematics which start at the pre-kindergarten level (USNY and NYSED, 2002). In fact, the current federal and state-level initiatives to promote rigorous pre-kindergarten mathematics education intend their main beneficiaries to be the children from low-SES families. Thus, to meet standards, these teachers may, for example, have had to adopt goal-based instruction for mathematics. On the other hand, since the teachers of middle-SES children worked at private schools, they may have been less strongly influenced by current focus on educational accountability and were able to maintain their traditional beliefs on early childhood education, which emphasize, for example, social development over academic learning.

Other factors involve the background experiences of the teachers, which differed depending on the SES of the children. As is typical for early childhood teachers around the country, teachers of low-SES children tended to be from a minority background, have a lower level of education, and less pre-kindergarten teaching experience than teachers of middle-SES children (Saluja et al., 2002; Whitebook et al., 1989).

One possibility is that the differences in educational level may influence teacher beliefs. As previous studies have shown (McMullen and Alat, 2002; Snider and Fu, 1990), teachers with extensive education may, for example, tend to acquire a strong preference for a child-centered view of valuing children's interests over teachers' goals. Another possibility is the difference in teaching experience between the two groups of pre-kindergarten teachers. The middle-SES teachers, with their longer experience in the culture of early childhood education, were more likely to have internalized the cautions and warnings against purposefully teaching mathematics to young children. A third possibility is the teachers' ethnic background. Johnson-Beykont (1999) showed that pre-school teachers with poor and minority family backgrounds tended to stress academic preparation significantly more than their White and/or higher-SES colleagues. Research further shows that shared ethnicity and SES with their students tends to help teachers to better understand the needs of their students (Ehrenberg et al., 1995).

As the present study was not designed to disentangle these factors as contributors to teacher beliefs, to investigate the relative contributions of the children, the school settings, and several aspects of teachers' own background experience and also to identify other possible factors to shaping teachers' pedagogical beliefs, further research is required to examine issues of causation.

implications for professional development for pre-kindergarten teachers

Until recently little stress has been placed on teaching mathematics at the pre-kindergarten level. As a result, overall teachers tended to have little experience with teaching mathematics, let alone purposefully reflecting on it (Copley, 2004; Ginsburg et al., 2006; Sarama and DiBiase, 2004). Some of the participating pre-kindergarten teachers commented on their somewhat careless attitude toward mathematics, especially when compared to literacy:

> Overall I'm feeling, I don't know much about teaching math. I know a little bit, you know, enough that, I know which materials to provide the children. But I really, you know, there are a lot of uh ... This has made me think a lot about different aspects of teaching math that I haven't really thought of before in preschool.

> Well, math, it's hard. I don't think we actually do as much math as we should. You know, now that we're doing this [interview], I realize we don't ... We do it. It's incorporated into everything that we do. But I don't think there's as much of a focus on it as there probably should be. There seems to be more reading, and writing, and knowing your letters, and colors.

This situation has begun to change, especially since many national and state organizations as well as localities stress the need for early mathematics education (Ginsburg et al., 2006).

How can we design effective continuing professional development for the pre-kindergarten teachers of low- and middle-SES children? Our study revealed that low- and middle-SES teachers tended to share some beliefs but not others; professional development should take this into account.

Both groups of teachers, most importantly, should be aware that young children are capable learners with powerful informal mathematics understanding who need and want to be intellectually challenged and stimulated (Ginsburg, 1999; Greenes, 1998). Teachers' over-anxiousness about pressuring or hurrying children should not prevent them from challenging curious young minds to extend children's mathematical understanding. Pre-kindergarten mathematics education, indeed, should be both broad and deep. The results of this study suggest that the pre-kindergarten teachers may need to expand their conception of mathematical content for young children to include all the major content areas, namely, number and operations, geometry, measurement, algebra, and data analysis/probability (NAEYC and NCTM, 2002; NCTM, 2000). They must have a deep understanding of core or big ideas in each of these five content areas and the connections and sequences among the big ideas to create a coherent, powerful, and challenging pre-kindergarten mathematics curriculum.

low-SES publicly funded pre-kindergarten teachers

The teachers of low-SES children, who were aware of the underachievement of children from poor and minority families, frequently stressed their strong commitment to pre-kindergarten mathematics education goals and plans in the classroom and at home in order to prevent their young students from getting on a trajectory of underachievement starting in kindergarten. However, they tended to be less vocal about how they would assist their students in learning and achieving these goals. That is, they appeared to have a strong sense of *where* they should go with their students in terms of pre-kindergarten mathematics, but not so much sense of *how* they could get there. These teachers need to understand that *how* children learn (or the *processes* of mathematics education) is as important as *what* they learn (or the *contents* of mathematics education). The teachers should be exposed to and become familiar with a range of developmentally appropriate instructional strategies and assessment methods, which support children in learning key mathematics contents through the processes of problem-solving, reasoning, communicating, connecting, and representing (NAEYC and NCTM, 2002; NCTM, 2000).

In addition, the teachers' discussion of appropriate instructional strategies centered mainly around a reliance on a theme-approach and ready-made curricula and materials. While the theme approach can be an efficient and effective way of weaving mathematics into other subject matters, experts (Bredekamp and Rosegrant, 1995; Copley and Padron, 1998; Hirsch, 1996) warn about the risk of it becoming 'a grab bag of any mathematics-related experiences that seem to relate to a theme or project' but do not provide a coherent educational experience (NAEYC and NCTM, 2002: 10). Also, while the use of ready-made mathematics curricula and materials may be helpful in terms of allowing children to encounter key mathematical concepts in a logical sequence, this approach needs to be combined with a deep understanding of pedagogy in order to avoid the risk of superficial teaching (Ryan, 2004; Walsh et al., 1993).

middle-SES private pre-kindergarten teachers

Unlike the teachers of low-SES children, teachers of middle-SES children tend to place mathematics on the backburner. These teachers may need to reevaluate the needs of their students and reconsider their relaxed attitudes towards pre-kindergarten mathematics. While it may be true that their students generally do better than their peers from low SES families, they should become aware that American students in general experience difficulties in learning school mathematics and that early mathematics education can provide them with a solid

affective and cognitive and affective foundation in mathematics (Bowman et al., 2001; NAEYC and NCTM, 2002; NCTM, 2000).

The middle-SES teachers preferred to introduce mathematical learning through children's play, exploration, discovery learning, and problem solving, particularly to foster children's positive dispositions towards mathematics. Free play and spontaneous exploration, as valuable as they can be, by themselves do not guarantee mathematical development; they offer rich possibilities. Thus, besides embedding mathematics learning in play, carefully planned and sequenced experiences that focus children's attention on a particular mathematical idea or set of related ideas are needed in order to promote the learning of concepts and skills as well as positive dispositions. Also, contrary to these teachers' beliefs, the effective use of computers provides not only a valuable opportunity for mathematical learning but also for social interaction (Clements, 1999; Haugland, 1999, 2000). Even regarding the computer use at home, teachers can help parents to select software programs that go beyond inappropriate drill and practice (Hohman, 1998).

As pre-kindergarten teachers engage in continuing professional development and gain experience with teaching mathematics to young children, their beliefs and pedagogy may improve – or should, if pre-kindergarten mathematics education is to be successful.

appendix A: sample vignette

vignette II

As for math education, teacher C feels that four-year-olds are ready for it, in fact, more than ready. The more she engages children in her class in math-related activities, the more she is surprised by their ability and motivation to learn. When young children are not ready for math, it only means that they have not been exposed to appropriate math experiences. The children in her class are engaged in math activities such as counting to 50, and addition and subtraction of concrete objects. Teacher C believes that the earlier children are introduced to math, the more and the better they will learn.

On the other hand, teacher D feels that early introduction to math is unnecessary, and even could be harmful to four-year-olds. She thinks that because four-year-olds are still immature, both mentally and physically, they are not ready for math education other than perhaps simple counting, shapes, and one-to-one correspondence. According to her, as time passes and children mature, it gets easier for them to learn, and most children eventually become math-literate. Therefore, there is no reason to hurry at the preschool level.

appendix B: dimensions and codes

Dimensions/codes ($r=$)[a]	Code descriptions
Strong focus	
Importance of math ($r=0.80$)	Teachers should place a high priority on children's learning of mathematics.
Specific time ($r=0.72$)	Teachers should set aside time specifically for math.
Can't catch up later	Children who are behind in their early years will have a difficult time in later schooling.
Kindergarten preparation ($r=0.71$)	Teachers should prepare children for kindergarten in order to get accepted and/or to be competitive with their kindergarten classmates.
Ready-made ($r=0.56$)	Materials/curriculum that are ready-made should be used.
Flexible	
Parental demands ($r=0.83$)	Some parents expect their children's outstanding school success and/or request more rigorous academics education including homework.
Postpone ($r=0.81$)	When children do not seem to be interested and/or ready, teachers should postpone the introduction of a math activity to some other time.
Flexible plans ($r=0.73$)	Teachers should be flexible and modify their math lesson plans or goals in response to children's needs.
Social development ($r=0.58$)	Teachers should focus on socialization with peers/adults and development of social competence such as sharing and negotiating.
Can catch up later ($r=0.44$)	Teachers should understand that children who are behind now will catch up later and they all will become literate in math eventually.
Goal-based	
Thematic approach ($r=0.74$)	Teachers should design a preschool curriculum based on a certain theme or topic and math activities should be organized around it.
Parent as teacher ($r=0.71$)	Parents should work with children at home to prepare and/or supplement school learning.
Encourage ($r=0.65$)	When the child(ren) show no interest in math, teachers should suggest or encourage them to participate and engage in math activities.

Goals/plans ($r = 0.57$)	Teachers should have overall goals/plans for math education.
Child-centered	
Informal assessment ($r = 0.79$)	Teachers should gather evidence about children's understanding and learning of math through classroom observations and interviews.
Content plans ($r = 0.78$)	Teachers should work with goals or plans to introduce children to certain math contents.
Individually appropriate ($r = 0.70$)	Teachers should adapt the math curriculum to each individual child's need and work with a child at his/her own level and pace.
Children's interest ($r = 0.61$)	Teachers should be aware of the interests of children in their class and make these the basis for the development of the curriculum.
Do not test ($r = 0.56$)	Teacher should not test, quiz or assess children's learning and development of math.
Arithmetic	
Numeracy readiness ($r = 0.86$)	Teachers should promote children's understanding of more and less, one-to-one correspondence, and/or simple counting.
Increased academic demands ($r = 0.83$)	Demands and pressure for early academics learning are increased by current educational trends such as higher standards and accountability advocated by boards of education, school districts, or school systems.
Direct/instruct ($r = 0.81$)	Teachers should direct math activities in which children are asked to listen to their instructions and follow their directions.
Numerals ($r = 0.70$)	Teachers should promote children's learning of names and shapes of numerals and/or their ability to recognize and write numerals.
Simple arithmetic ($r = 0.63$)	Teachers should promote children's understanding of addition/subtraction and/or engage children in counting activities that go up to 20 or more.
Child-initiated	
No ready-made ($r = 0.90$)	Materials/curricula that are ready-made including workbooks, worksheets or pre-drawn/precut materials should not be used.

No preparation ($r = 0.85$)	Teachers do not need to prepare children for kindergarten or future schooling.
Explore/discover ($r = 0.82$)	Children should learn on their own through self-initiated play or explorations and discoveries.
Positive feelings/ dispositions ($r = 0.80$)	Teachers should promote positive dispositions and feelings towards school/learning or math such as a positive mind-set for school or math, curiosity, desire to learn in children.
Problem-solving ($r = 0.77$)	Teachers should build new mathematical knowledge through engaging children in solving problems.
Enjoyable	
Emotional development ($r = 0.81$)	Teachers should focus on emotional well-being such as a sense of security and development of positive self-esteem, confidence, and independence.
Small group ($r = 0.80$)	Teachers should conduct a math activity with individuals or a small group of children or not at circle time or a large group of children.
Appeal ($r = 0.65$)	Teachers should design a math activity using things such as topics/toys in which the child(ren) are interested in order to promote their interest and learning in math.
Fun ($r = 0.51$)	Teachers should ascertain that a math activity is fun and promotes feelings of joy, pleasure, amusement, and excitement in children.
Non-pressured	
Do not hurry ($r = 0.80$)	Teachers should not push children to accelerate their learning beyond their own pace or to catch up with their peers.
Different ability ($r = 0.77$)	As individuals, four-year-olds learn and develop at different rates.
Not stressful ($r = 0.71$)	Teachers should ascertain that a math activity does not provoke negative feelings such as anxiety, stress, pressure, intimidation, or competition.
Emergent curriculum ($r = 0.62$)	Teachers should help children learn math at times when they show interest.
Unready & immature ($r = 62$)	Four-year-olds are immature and/or incompetent to understand and learn math.
Different aptitude ($r = 0.55$)	As individuals, four-year-olds have different interests and aptitudes and/or some four-year-olds in my class are particularly interested in math.

No computers	
No classroom computer ($r = 0.90$)	Computers should not be used as a learning tool for math in classrooms.
Classroom computer ($r = -85$)	Computers should be used as a learning tool to promote children's understanding and learning of math in preschool classrooms.
Home computer ($r = 0.77$)	All or most four-year-olds in my class use computers sufficiently at home and/or they are skilled at operating computers.
Enriched home environment ($r = 0.54$)	Most four-year-olds in my class have rich stimulation at home that promotes their ideal development and learning.
Anti-social computer ($r = 0.52$)	Computers have a negative effect on children's social development and/or prevent children's interactions with peers/adults.

[a] Correlation of the code with its dimension.

note

1. The data used in this study were collected as a part of a larger study on pre-kindergarten teachers' beliefs about appropriate literacy and mathematics education (Lee and Ginsburg, in press). The current study was undertaken to re-examine the data specifically on mathematics and to provide a richer and more detailed description of the pre-kindergarten teachers' beliefs about this subject matter.

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